This problem comes from Griffiths' Introduction to Quantum Mechanics. It's problem 2.1a.

***Problem 2.1** Prove the following theorems:

(a) For normalizable solutions, the separation constant E must be *real.* Hint: Write E (in Equation 2.6) as $E_0 + i\Gamma$ (with E_0 and Γ real), and show that if Equation 1.20 is to hold for all t, Γ must be zero.

The wave function

$$\Psi(x,t) = \psi(x)e^{-iEt/\hbar} \tag{[2.6]}$$

Assuming E is complex.

$$E = E_0 + i\Gamma$$

We normalize our wave function.

$$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx = 1 \tag{[1.20]}$$

$$|\Psi(x,t)|^2 = \psi(x)^2 e^{-2iEt/\hbar}$$

Since we are integrating by x and not t, the whole right side comes out of the integral.

$$e^{-2iEt/\hbar} \int_{-\infty}^{+\infty} \psi(x)^2 dx = 1$$

I'm pretty sure this is already wrong because I'm not doing the $|\Psi|$. That is.. yeah instead of multiplying it by itself I should be multiplying by the complex conjugate. So let's get some computer algebra to help us along with this.

Using Gemini: see normalize_gemini1.md

"When you square the absolute value of a complex number, $|z|^2 = z * conjugate(z)$, where $conjugate(z)$ is the complex conjugate of z (obtained by changing the sign of the imaginary part)."

So I was right about that.

Gemini and maxima agree that $abs(Psi(x,t))^2 = psi(x)^2$ but I am now confused.

So now we integrate.

$$\int psi(x)^2 = 1$$

Nowhere here do we see E. So we're stuck. So we ask gemini.

Do you agree with the statement "For normalizable solutions, the separation constant E must be real."? Why?

Okay Gemini is repeatedly contradicting itself, so let's go through it's final logic.

$$E = E_r + iE_i$$
$$e^{-iEt/\hbar} = e^{-i(E_r+iE_i)t/\hbar}$$
$$= e^{-iE_rt/\hbar}e^{-i^2E_it/\hbar}$$
$$= e^{-iE_rt/\hbar}e^{E_it/\hbar}$$
$$|e^{-iEt/\hbar}|^2 = e^{-iEt/\hbar} * conjugate(e^{-iEt/\hbar})$$
$$conjugate(e^{-iEt/\hbar}) = e^{iE_rt/\hbar}e^{E_it/\hbar}$$
$$|e^{-iEt/\hbar}|^2 = e^{-iE_rt/\hbar}e^{E_i)t/\hbar}e^{iE_rt/\hbar}e^{E_it/\hbar}$$

So now we can simplify our answer here.

$$|e^{-iEt/\hbar}|^2 = e^{-iE_rt/\hbar}e^{E_it/\hbar}e^{iE_rt/\hbar}e^{E_it/\hbar}$$
$$= e^{-iE_rt/\hbar+E_it/\hbar+iE_rt/\hbar+E_it/\hbar}$$
$$= e^{E_it/\hbar+E_it/\hbar}$$
$$= e^{2E_it/\hbar}$$

Now it's possible to confidently solve the problem. With a bit of logic that Gemini used (that we understood but obviously it's not completely honest to say that this thought is original when Gemini said it first), we can explain the nature of the normalization.

$$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx = 1 \qquad ([1.20])$$

$$= e^{2E_it/\hbar} \int_{-\infty}^{+\infty} \psi(x)^2 dx = 1$$

No matter what our ψ is, setting $t = \infty$, we get

$$= e^{2E_i\infty/\hbar} \int_{-\infty}^{+\infty} \psi(x)^2 dx = 1$$

The only legitimate answer to this where E_i is not zero is where $\psi(x)$ cancels out the infinity it is being multiplied to. I am not aware of any non-zero solution and since we know that the whole thing has to be equal to 1, we have a pretty clear solution to this problem.

$$E_i = 0$$